Space-efficient, Fast and Exact Routing in Time-dependent Road Networks

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ABSTRACT

We study the problem of computing shortest paths in massive road networks with traffic predictions. Existing techniques follow a two phase approach: In an offline, preprocessing step, a database index is built. The index only depends on the road network and the traffic patterns. The path start and end are the input of the query phase, in which shortest-paths are computed. All existing techniques have a large index size, slow query running times, or may compute suboptimal paths. In this work, we introduce CATCHUp (Customizable Approximated Time-dependent Contraction Hierarchies through Unpacking), the first algorithm that simultaneously achieves all three objectives. We perform an extensive experimental study on a set of real world instances and compare our approach with state-of-the-art techniques. Our approach achieves up to 30 times smaller indexes than competing approaches. Additionally, our index can be updated within a few minutes if traffic patterns change.

1 INTRODUCTION

Routing in road networks is a well-studied topic with a plethora of real world applications. The core problem setting consists of computing a fastest route between a source and a target. The idealized problem can be formalized as the classic shortest path problem. Streets are modeled as arcs. Street intersections are modeled as nodes. Travel times are modeled as scalar arc weights. Unfortunately, this idealized view does not model certain important real world effects. An important example are recurring commuter congestions. In this paper, we therefore consider an extended problem setting in which arc weights are time-dependent. The weight of an arc is a function of the moment where a car enters the arc. Figure 1 depicts an example.

Web-based routing services such as Google, Baidu, Yandex, Bing, Apple, or HERE Maps are ubiquitous. They all share a similar setup in which a small set of servers answers queries of many users. Computing routes using Dijkstra's [12] algorithm is possible. However, the large query running times of several seconds per path would require a huge number of servers. This is cost prohibitive.

To reduce the number of servers and to enable interactive applications, it is crucial to answer queries quickly. To achieve small query running times, a two-phase approach is used. In the first phase, the preprocessing, a database index is constructed. The index only depends on the road networks and the time-dependent arc weights. In the second phase, shortest paths are computed by querying this database index. Beside reducing the number of servers, it is also desirable to have small index size. This reduces the costs per server.

Contraction Hierarchies (CH) are a popular and effective technique to construct such indexes. They were introduced in [15] and

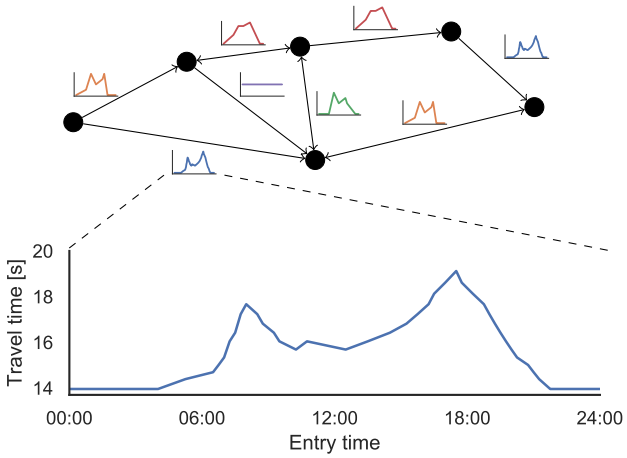

Figure 1: The graph of a small road network with predicted travel times for each road segment.

refined in [11]. The core idea is to introduce additional arcs called *shortcuts*. Shortcuts replace parts of input graph like in Figure 2. They allow bypassing these parts of the input graph. When computing shortest paths, a small number of shortcuts are explored instead of a large number of arcs in the input graph. This approach achieves speedups of several orders of magnitude over Dijkstra's algorithm. However, the implementations of [11, 15] use scalar weights and do not solve the time-dependent setting.

An adaptation for the time-dependent setting was made in [2]. The implementation uses periodic piecewise linear functions to model travel times. The functions are represented as a sequence of *breakpoints*. [2] attaches a function to every shortcut. Let s be a shortcut and P the path bypassed by s. The travel time along s must be equal to the travel time along P, i.e., s's travel time function is the composition of the functions along the arcs of P. Unfortunately, the number of breakpoints in s's function corresponds to the accumulated number of breakpoints of the composing functions. Shortcuts aggregate the complexity of paths they represent, rather than bypassing it. This leads to slower preprocessing and prohibitive memory consumption. In [2], solutions based on function approximations are presented. Unfortunately, they either lead to slower or inexact queries. To make time-dependent CHs really tractable, a better function compression schema is crucial. Such a schema is the core contribution of our paper.

We observe that the shortest path between two nodes changes less frequently over time than the travel time between the nodes.

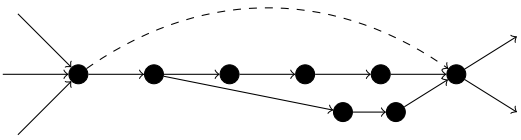

Figure 2: A shortcut arc bypassing several nodes.

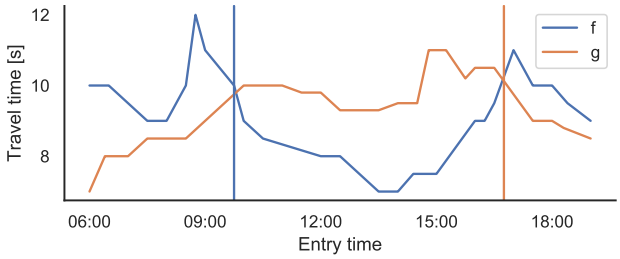

Figure 3: Travel time functions for two different paths between the same start and end node.

Oftentimes, it does not change at all. For example, going via a highway may be slow due to congestion but is usually still the fastest option. Consider the functions f and g in Figure 3. Both functions have a significant number of breakpoints. The function $h = \min(f, g)$ has a similar number of breakpoints as f and g. If we store h explicitly, we need memory proportional to the number of breakpoints of f and g. However, we do not store h explicitly. Instead, we store the time intervals in which f is smaller than g. In the example, we store that from 6:00 on g wins until 9:45. From this time on f wins until 16:45. After this time, g wins again. There are significantly fewer time points in which h switches from f to g and vice versa, than there are breakpoints. We therefore have a more compact representation.

Formulated differently, we store in the algorithm base variant at each shortcut s only the information necessary to unpack s into the corresponding input path P. Using this information, a path that uses shortcuts can be translated into a path that only uses input graph arcs. In the input graph, the travel time along the whole path can be computed by iteratively evaluating the travel times of each input arc. Evaluating the travel time along a path is faster than executing Dijkstra's algorithm on the input graph. Our query algorithm is a modified CH query that unpacks shortcuts on-demand within the search space. To allow further pruning, we store the minimum and maximum travel times of each shortcut.

While this compression schema works well for query computations, we need to maintain approximated shortcut travel time functions during index construction. Fortunately, the index construction can be done on one expensive server. Afterwards, the index can be distributed to a set of inexpensive query servers.

Related Work. Routing in road networks has been extensively studied in the past decade. An overview over the field can be found in [1]. Here, we focus on speed-up techniques for time-dependent road networks.

Several time-independent speed-up techniques have been generalized to the time-dependent setting. ALT [17], an approach using landmarks to obtain good A* [21] potentials has been generalized to TD-ALT [27] and successively extended with node contraction to TD-CALT [9]. Even when combined with approximation, TD-CALT queries may take longer than 10ms on continental sized graphs. SHARC [4], a combination of ARC-Flags [26] with shortcuts which allows unidirectional queries was also extended to the time-dependent scenario [7]. It can additionally be combined with ALT yielding L-SHARC [7]. SHARC can find short paths in less than a millisecond but does not always find a shortest path. MLD/CRP [8, 22] has been extended to TD-CRP [5] which can be used in a time-dependent setting.TD-CRP requires approximation to achieve reasonable memory consumption. TD-CRP also does not always find a shortest path. Another approach is FLAT [24] and its extension CFLAT [25]. CFLAT features sublinear query running after subquadratic preprocessing and guarantees on the approximation error. Unfortunately, preprocessing takes long in practice and generates a prohibitively large index size.

There are several approaches based on CHs [15]. Three were introduced in [2]: Time-dependent CHs (TCH), inexact TCHs, and Approximated TCHs (ATCH). TCHs achieve great query performance but at the cost of a huge index size on state-of-the-art continental sized instances. Index size can be reduced at the cost of exactness (inexact TCHs) or query performance (ATCHs). An open-source reimplementation of [2] named KaTCH[1] exists. A simple heuristic named Time-Dependent Sampling (TD-S) was introduced in [30]. It samples a fixed set of scalar values from the time-dependent functions. It has a manageable index-size and fast query times but does not always find a shortest path.

Contribution. We present CATCHUp – Customizable Approximated Time-dependent Contraction Hierarchies trough Unpacking, a time-dependent generalization of Customizable Contraction Hierarchies [11] and a thoroughly engineered implementation. Our implementation achieves fast and exact queries with performance competitive to TCH queries while requiring up to 30 times less memory. Preprocessing is split into a metric independent phase and a metric dependent customization phase. We describe several optimizations to the customization necessary to achieve good customization performance in the time-dependent scenario. We also present algorithms which allow us to employ approximation during the customization without sacrificing exactness. This allows us to update the metric within a few minutes on modern production-grade continental sized instances. Finally, we propose an improved ATCH query algorithm. Both the query algorithm and some of the customization optimizations could also be employed by (A)TCHs.

The rest of this paper is organized as follows. In Section 2 we introduce some notation and basic algorithmic ingredients. Section 3 covers our shortcut data structure and several operations on shortcuts. We describe our preprocessing and query algorithms in Section 4. In Section 5 we discuss our experimental evaluation and the performance of our implementation. We conclude in Section 6.

[1] https://github.com/GVeitBatz/KaTCH

2 PRELIMINARIES

We model road networks as directed *graphs* $G = (V, A)$. A *vertex* $v \in V$ represents an intersection and an *arc* $(u, v) \in A$ represents a road segment. Let $n = |V|$ denote the number of nodes and $m = |A|$ the number of arcs. A path is a sequence of vertices $[v_1, ..., v_k]$ such that $(v_i, v_{i+1}) \in A$.

Every arc a has a *travel time function* $f_a : \mathbb{R} \to \mathbb{R}^{>0}$ mapping departure time to travel time. We also refer to these functions as *travel time profiles*. Similar to competing approaches, we assume that travel time functions fulfill the *FIFO property*, that is for any $\sigma, \tau \in \mathbb{R}$ with $\sigma \leq \tau$, $\sigma + f(\sigma) \leq \tau + f(\tau)$ has to hold. Informally, this means that it is never possible to arrive earlier by starting later. If there are arcs that do not fulfill the FIFO property, the shortest path problem becomes $\mathcal{NP}$-hard [28]. In our implementation, travel time functions are periodic piecewise linear functions represented by a sequence of *breakpoints*. We denote the number of breakpoints by $|f|$. However, our ideas are agnostic to the representation of travel time functions and will work as well with other function classes supporting the following operations:

Most essential, we require an Eval operation which calculates the value $f(\tau)$ of a function f at time τ. We extend Eval to paths by evaluating the arcs successively:

$$\textsc{Eval}([v_1, v_2, ..., v_i], \tau) = \textsc{Eval}(f_{(v_1,v_2)}, \tau) + \textsc{Eval}([v_2, ..., v_i], \tau + \textsc{Eval}(f_{(v_1,v_2)}, \tau))$$

We also require a Union operation which combines two functions with disjoint domains, for example two different segments of a piecewise linear function:

$$\textsc{Union}(f, g)(\tau) = \begin{cases} f(\tau) & \text{if } \tau \in \text{domain}(f) \\ g(\tau) & \text{else} \end{cases}$$

To chain the travel time functions of two consecutive arcs, we define the Link operation. Given two travel time functions f and g for arcs (u, v) and (v, w), the linked travel time function has to fulfill the following property: $\textsc{Link}(f, g)(\tau) = f(\tau) + g(f(\tau) + \tau)\ \forall \tau \in \mathbb{R}$. Then, $\textsc{Link}(f, g)$ is the travel time function of the path $[u, v, w]$. To combine different travel time functions for paths with the same start and end, we define the Merge operation. For two travel time functions of different $[u, ..., v]$ paths, the merged travel time functions consists of the faster travel time at each point in time: $\textsc{Merge}(f, g)(\tau) = min(f(\tau), g(\tau))\ \forall \tau \in \mathbb{R}$. The result of $\textsc{Merge}(f, g)$ is the minimum travel time function between u and v on the given paths.

Both linking and merging can be implemented by simultaneously sweeping over the breakpoints of both functions. Periodic piecewise linear functions fulfilling the FIFO property are closed under linking and merging. A linked travel time function $link(f, g)$ has at most $|f| + |g|$ breakpoints. A merged travel time function $merge(f, g)$ can contain one breakpoint per intersection of segments of the input travel time functions and breakpoints from the input travel time functions which is in $O(|f| + |g|)$.

Given a departure time τ and vertices s and t, an earliest-arrival query asks for the minimum time to travel from s to t starting from s at τ. Such a query can be handled by Dijkstra's algorithm [12] with little modifications [14]. The algorithm keeps track of the earliest known arrival time ea_v at each vertex v. These labels are initialized with τ for s and ∞ for all other vertices. The queue is initialized with s. In each step the next vertex u with minimal earliest arrival ea_u is extracted from a priority queue and all outgoing arcs are relaxed. To relax an arc (u, v), the algorithm checks if $\text{ea}_u + f_{(u,v)}(\text{ea}_u) < \text{ea}_v$ and updates the earliest arrival and the queue position of v if the arrival time can be improved. When nodes are popped from the queue, their earliest is final. This property is denoted as *label-setting*. Once t is extracted from the queue, we know the final earliest arrival at t. We refer to this algorithm as *TD-Dijkstra*.

The *A* algorithm* [21] is an extension to Dijkstra's algorithm. It reduces the size of the search space by guiding the search towards t. Each node u has a potential $\rho_t(u)$ which is an estimate of the distance to t. The priority queue is then ordered by $\text{ea}_u + \rho_t(u)$.

Contraction Hierarchies [15] are a speed-up technique for road networks. All nodes are ranked by some measure of importance. Nodes with higher rank should be part of more shortest paths. During preprocessing, all nodes are contracted in ascending order of their importance. Contracting a node v means conceptually removing it from the network but preserving all shortest distances among the remaining nodes. This is achieved by inserting *shortcut arcs* between the neighbors of v if a shortest path between them goes over v. A single shortcut always bypasses exactly one node but the arcs between the contracted node and its neighbors may also be shortcuts. Thus, recursively unpacking shortcuts may yield much longer bypassed paths. The result of the preprocessing are two augmented graphs $(G^\wedge, G^\vee)$, also called a Contraction Hierarchy. $G^\wedge$ contains only outgoing arcs to higher ranked nodes. $G^\vee$ contains only incoming arcs from higher ranked nodes. Shortest-path queries can be answered by performing a bidirectional Dijkstra search where the forward search runs on $G^\wedge$ and the backward search on $G^\vee$. The construction guarantees that this algorithm will find a shortest up-down-path which has the correct length.

Contraction Hierarchies use the metric to determine the importance of each node. If the metric changes, the entire hierarchy needs to be recomputed which is computationally expensive. *Customizable Contraction Hierarchies* (CCHs) [11] avoid this by splitting the preprocessing into two phases. In the first metric-independent phase, the nodes are contracted based on a nested dissection order [3, 16]. Since no metric is available, all possible shortcuts are inserted. In the second phase, called customization, a metric is applied to the graph and the weights of all shortcuts are determined by a bottom-up sweep over all arcs of the graph. The result is a valid Contraction Hierarchy and the same query algorithm as for classical CHs can be employed.

3 SHORTCUT DATA STRUCTURE

In a non time-dependent setting, only a single number is associated with each shortcut. It represents the weight of the entire path bypassed by the shortcut. Applying the same idea to the time-dependent setting would mean storing the travel time profile of the shortcut. But this is very expensive in terms of memory consumption. The shortcut profile will usually contain about as many breakpoints as all original arcs in the represented path combined. Rather than explicitly storing this profile, we only store information to unpack the shortcut to the best path for each point in time and a static upper and lower bound.

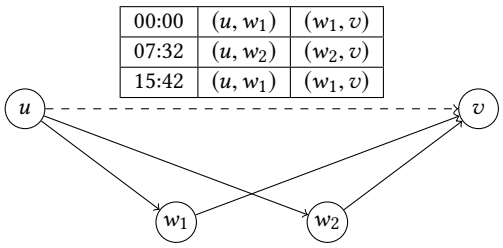

Figure 4: A shortcut with associated time-dependent unpacking information

Formally, for each shortcut arc $s = (u, v)$ we have a lower bound $\underline{b}_s$, an upper bound $\overline{b}_s$ and set of time-dependent *expansions* X_s for unpacking. For an expansion $x \in X_s$, we denote the time during which x represents the shortest path as the *validity interval* Π_x of x and the lower node of the triangle $[u, w_x, v]$ as w_x. Since a shortcut always expands to a path of length two, knowing the middle node is sufficient for unpacking. In our implementation, the expansion information is represented as an array of triples $(\tau, (u, w), (w, v))$. τ is the beginning of the validity interval and (u, w) and (w, v) are arc ids. This information can be stored in 16 bytes for each entry – 8 bytes for the timestamp and 4 bytes for each arc id. Figure 4 depicts an example of a shortcut with expansion information. A shortcut could also represent an original arc or no arc at all during a certain time interval. Both these cases are represented as special arc id values.

Algorithm 1: Unpack

Input: Shortcut $s = (u, v)$ with unpacking data X_s, Time τ
Output: Unpacked path $[u, ..., v]$

$x_\tau \leftarrow x \in X_s$ such that $\tau \in \Pi_x$ `// binary search`

if x_τ *represents original arc* $(u, v) \in A$ **then**
  return $[u, v]$
else
  $p \leftarrow$ Unpack$((u, w_x), \tau)$
  return $p \cup$ Unpack$((w_x, v), \tau +$ Eval$(p, \tau))$

There are several operations which require us to unpack a shortcut s: evaluating the travel time $f_s(\tau)$ of a shortcut, marking the unpacked original arcs as contained in a search space or constructing the travel time profile f_s. Algorithm 1 depicts how to achieve this. To unpack a shortcut at a point τ in time, we first need to determine the relevant expansion x such that $\tau \in \Pi_x$. This can be done performing a binary search in X_s. We then recursively apply the operation to (u, w_x) at τ and to (w_x, v) at $\tau + f_{(u,w_x)}(\tau)$. The recursion stops when reaching original non-shortcut arcs. Evaluating the travel time of the first arc is always necessary to determine the correct point in time for the second arc. The travel time of a shortcut can be evaluated by unpacking the path and then evaluating the travel times of the original arcs successively. When a shortcut is unpacked only for travel time evaluation, the unpacking can be skipped if the upper and lower bounds are equal. In that case the travel time of the shortcut is constant.

Constructing the travel time profile of a shortcut is conceptually similar and shown in Algorithm 2. We recursively unpack shortcuts until we reach arcs of the original graph where exact travel time functions are available. We may need to unpack several different expansions for different times and combine them with the Union operation. For each expansion we check if its validity overlaps with the time range for which we want to construct the profile. If so, we retrieve the profile for the first arc during this overlap. Then, we calculate the profile for the second arc during the overlap shifted using start and end of the profile of the first arc. Both profiles are then linked and appended to the profile of the shortcut.

Implementing this algorithm naively may cause performance issues since many memory allocations are performed for intermediate results. We avoid this by keeping all intermediate results in two buffers which are reused for all invocations of this algorithm. The buffers are stored as vectors and can grow dynamically but will never shrink. Once they have grown to an appropriate size, no more memory allocations will be necessary. Each buffer can contain many travel time functions stored consecutively. The link operation will read the last two functions from one buffer and append the result to the other buffer. Then, the two input functions will be truncated from the first buffer. After swapping, the buffers can be used again for the next link operation. The same schema can be employed for taking the union of partial functions.

Linking two shortcuts (u, w) (w, v) is as simple as creating a new shortcut s with $\underline{b}_s := \underline{b}_{(u,w)} + \underline{b}_{(w,v)}$, $\overline{b}_s := \overline{b}_{(u,w)} + \overline{b}_{(w,v)}$ and $X_s := \{(0, (u, w), (w, v))\}$. Merging is more complicated. To merge two shortcuts r and s we need to obtain the travel time profiles f_r and f_s using the UnpackProfile operation. By merging the actual profiles f_r and f_s, we obtain the times Π_r and Π_s in which each of them is shorter. The combined expansions contain the expansions from s during the time when s is shorter and the ones from r when r is shorter: $X_{min(r,s)} := \{(\Pi_r \cap \Pi_x, w_x) \mid x \in X_r\} \cup \{(\Pi_s \cap \Pi_x, w_x) \mid x \in X_s\}$. The combined bounds are each the minimum of the bounds of both shortcuts.

Algorithm 2: UnpackProfile

Input: Shortcut $s = (u, v)$ with unpacking data X_s, Time interval $[\tau, \pi]$
Output: Exact profile $f_s|_{[\tau,\pi]}$

Initialize f_s as function with empty domain
for $x \in X_s$ **do**
  $[\tau_x, \pi_x] \leftarrow [\tau, \pi] \cap \Pi_x$
  if x *represents original arc* $(u, v) \in A$ **then**
    $f_x \leftarrow f_{(u,v)}|_{[\tau_x,\pi_x]}$
  else
    $f_{(u,w_x)} \leftarrow$ UnpackProfile$((u, w_x), [\tau_x, \pi_x])$
    $f_{(w_x,v)} \leftarrow$ UnpackProfile$((w_x, v),$
     $[\tau_x + f_{(u,w_x)}(\tau_x), \pi_x + f_{(u,w_x)}(\pi_x)])$
    $f_x \leftarrow$ Link$(f_{(u,w_x)}, f_{(w_x,v)})$
  $f_s \leftarrow$ Union(f_s, f_x)
return f_s

4 ALGORITHMS

In this section, we describe our algorithms and implementation. We extend CCHs into an efficient approach for time-dependent routing using the shortcut data structure described in the previous section. CCHs follow a three-phase approach. The first phase is a metric-independent preprocessing phase. In the second phase, called customization, metric dependent preprocessing will be performed. Finally, in the third phase, shortest-path queries can be answered. We will cover each phase in turn and first describe the classical CCH algorithms before presenting our adaptation.

4.1 Metric-Independent Preprocessing

Metric-independent preprocessing is the same as for CCHs [11]. Two steps are necessary. First, we need a measure of importance for each node. In CCHs, this is done using a nested dissection order. For that, a small separator is searched. Removing the separator splits the network into two (or more) disconnected components. The nodes in the separator get the highest ranks assigned. The process is then applied recursively to each of the disconnected components. We use InertialFlowCutter [18], a recently proposed combination of InertialFlow [29] and FlowCutter [20], to calculate the order.

Second, we need to compute the CH and insert shortcuts into our graph. This is done by iteratively contracting nodes from lowest to highest rank. Contracting a node means removing it from the graph and inserting shortcuts between its neighbors unless they are already connected. Since no metric is available, all potential shortcuts must be inserted. The neighbors of each contracted node will become a clique.

We implement a slightly modified contraction algorithm first presented in [31] inspired by a linear-time graph chordality check [19]. When contracting a node, we do not connect all neighbors with each other. Rather, we just insert all neighbors of the current node into the neighborhood of its lowest ranked neighbor. The end result is the same. This algorithm can be implemented to run in linear time in the size of the output graph.

4.2 Customization

During customization, the weights of all shortcuts need to be computed. Shortcut weights are initialized to infinity. Shortcuts always bypass paths over exactly one node (see Figure 4). To calculate the weight of a shortcut, we need to find the shortest among those bypassed paths. That is, to calculate the weight of shortcut (u, v) we need find the shortest of all *lower triangles* $[u, w, v]$ where w has lower rank than u and v. This yields a correct Contraction Hierarchy if the weights of the arcs (u, w) and (w, v) are always final, before the weight of (u, v) is determined. This can be achieved by processing arcs by their lower ranked endpoint. For each arc all lower triangles will be enumerated, and the arc gets the weight of the shortest triangle assigned.

In our customization phase, the bounds and the unpacking information need to be computed for each shortcut. We apply the same basic schema as in [11] and iterate over all shortcuts in a bottom-up fashion. The shortcuts are traversed in order of their lower ranked node. For each shortcut we enumerate all lower triangles. For each triangle, we need to determine, if it is the shortest path at certain points in time and if so add it to the shortcut's expansion information. This requires us to obtain the travel time profile of the triangle through linking and merge it with the shortcut's travel time profile. Since the linking and merging operations are quite expensive, we implement several optimizations which we describe in this section.

Avoiding Unnecessary Operations. Many link and merge operations can be avoided by first checking the operation's necessity using the bounds. Suppose we have a triangle $[u, w, v]$ and a shortcut $s = (u, v)$. Before linking $f_{(u,w)}$ and $f_{(w,v)}$, we check if $\overline{b}_s \leq \underline{b}_{(u,w)} + \underline{b}_{(w,v)}$. If so, the linked path would be dominated by the shortcut, and we can skip linking and merging completely. If not, we link $f_{(u,w)}$ and $f_{(w,v)}$ and obtain $f_{[u,w,v]}$. We still can skip merging if one function is strictly smaller than the other, that is either $\overline{b}_s \leq \min(f_{[u,w,v]})$ or $\max(f_{[u,w,v]}) \leq \underline{b}_s$. Even if the bounds overlap, one function might still dominate the other. To check for this case, we simultaneously sweep over the breakpoints of both functions, determining the value of the respectively other function by linear interpolation. Only when this check fails, we perform the actual merge operation.

Sorting Lower Triangles. In non-time-dependent CCHs the work performed per triangle is very small. Two numbers need to be added and compared to a third which may be updated. In the time-dependent case performing linking and merging is much more expensive and it can be beneficial to perform some extra work to avoid as many link and merge operations as possible. To this end, we process the lower triangles $[u, w, v]$ of each arc in a specific order by first sorting them ascending by $\underline{b}_{(u,w)} + \underline{b}_{(w,v)}$. This way, shorter triangles are processed first and most longer triangles can be skipped completely due to the bound checks.

Static Precustomization. Due to the metric-independent construction, CCHs may contain more shortcuts than actually necessary. CHs avoid this problem by performing witness searches. Unnecessary shortcuts in CCHs can be removed by performing a so called perfect customization. During the perfect customization, shortcuts are processed in a top down order. For each shortcut (u, v), all triangles $[u, w, v]$ where w is either the highest ranked or middle ranked node are enumerated. If the weight of $[u, w, v]$ is less or equal to the weight of (u, v), the shortcut is not required and can be marked for removal. After all shortcuts have been processed, the marked shortcuts can be removed.

To establish preliminary bounds and to remove unnecessary shortcuts, we first perform a precustomization using only upper and lower bounds. Having processed all shortcuts bottom-up, we make a copy of each shortcut's lower bound $\underline{b}'_s$. Now, we process all shortcuts top-down, merging them with upper and intermediate triangles. Afterwards, a shortcut s can be removed if $\overline{b}_s < \underline{b}'_s$ because a shorter path than s exists via upper or intermediate triangles.

We keep the preliminary bounds obtained by the precustomization since they allow skipping additional link and merge operations. This requires some additional care when updating bounds during the customization with the exact travel time functions. As before, we update upper bounds by taking the minimum of the current upper bound and the new upper bound. Note that a shortcut might temporarily have a profile with values greater than the upper bound stored at the shortcut. However, updating lower bounds like this

would have no effect. The lower bound obtained during precustomization is guaranteed to be smaller or equal to any actual lower bound a function on the shortcut might have. But this bound is not necessarily as tight as possible. After the final profile for a shortcut is obtained, we can update the lower bound to the maximum of the lower bound from the precustomization and the actual profile.

Reusing Intermediate Results. Merging shortcuts requires that we know their travel time profiles. While it is possible to calculate the profiles by unpacking the shortcuts and linking the involved original arcs, we can get around this by reusing the results of previous merge operations. The order of the shortcut iteration guarantees that all shortcuts in the lower triangles have already been processed. We keep the results of these previous merge operations. Only when all shortcuts with (u, v) where u is the lower ranked node have been processed, we remove all intermediate results from shortcuts (w, u) where u is then the higher ranked node. This way, we do not need to unpack any shortcuts during the customization.

Parallelization. We employ both loop based and task based parallelism to distribute the workload of the customization among several cores. The original CCH publication [11] suggests processing shortcuts in levels. The level of a shortcut is determined based on the rank of its lower ranked node. All shortcuts of each level can be processed in parallel. We employ this approach to process arcs in the top-level separators. But since this approach requires a synchronization step on each level and may lead to bad load balancing, we switch to different strategy whenever possible. In [6] a task based parallelism approach utilizing the separator decomposition of the graph is described. Each task is responsible for a subgraph G'. Removing the top-level separator in G' decomposes the subgraph into two or more disconnected components. For each component a new task is spawned to process the shortcuts in the component. After all child tasks are completed, the shortcuts in the separator are processed utilizing the loop based parallelization schema. If the size of subgraph G' is below a certain threshold the task processes the shortcuts in G' directly. We use $n/(\alpha \cdot c)$ as the threshold with c being the number of cores and tuning parameter α set to 32 as suggested by [6].

Approximation. As we process increasingly higher ranked shortcuts during customization, the shortcuts represent increasingly longer paths and the associated travel time profiles become more and more complex. This leads to two problems. First, the running time of linking and merging scales with the complexity of the input function. Profiles with thousands of breakpoints lead to slow customization times. Second, storing these functions for later reuse – even though it is only temporary – consumes a lot of memory.

This is a problem which all hierarchical speed-up techniques for time-dependent road networks have to cope with. TCHs [2] partly mitigates this by optimizing the order towards contracting nodes with shortcuts with low profile complexity first. Since we use a predetermined nested dissection order, this is not possible for us. Additionally, during contraction TCHs swap out shortcuts profiles of already contracted nodes to the disk to reduce memory usage. The most common approach is to reduce the complexity of shortcut profiles by approximating them. This approach is also employed by TD-CRP and in ATCHs.

ATCHs apply approximation only after the contraction is completed to reduce memory usage during query time. This has the advantage that the approximation error on each profile function is known and that the shortcut unpacking information is exact. This allows ATCHs to perform exact queries despite the approximated travel time functions. The downside is that contraction performance still suffers from the shortcut complexity. TD-CRP on the other hand applies approximation during the customization. This solves the performance issues but only at the cost of exactness and unbounded[2] errors. Our approach works around these issues and applies approximation during the customization but still obtains exact shortcut unpacking information, which allows us to perform exact queries.

When approximating, we do not store one approximated function but two – a lower bound function and an upper bound function. These two replace the exact profile stored for later merge operations and will also be dropped when no longer needed. When linking approximated functions, we can link both lower and both upper bound functions. Linking two lower bounds yields a valid lower bound function of the linked exact functions because of the FIFO property. The same argument holds for upper bounds.

Merging approximated shortcuts is slightly more complicated. Our goal is to determine the exact unpacking information for each shortcut. We cannot reuse our bound functions as they are only approximations. Thus, we need to unpack shortcuts and reconstruct their exact travel time functions. But we can use the approximations to avoid unpacking the exact functions for times where no intersections can occur. To identify these parts, we merge the first function's lower bound with the second function's upper bound and vice versa. Where the bounds overlap, an intersection might occur. We then unpack the exact functions in the overlapping parts and perform the exact merging. To obtain upper and lower bounds of the merged function, we merge both lower bounds and both upper bounds.

To perform the actual approximation, ATCH and TD-CRP make use of the algorithm of Imai and Iri [23]. Given a maximum error bound ϵ, this algorithm can compute in linear time the piecewise linear function with the minimum number of breakpoints within the given error bound. The errors may be one-sided for the computation of bound functions. The great theoretic guarantees of this algorithm come at the cost of considerable implementation complexity and high constant runtime factors. We implemented this algorithm but also the much simpler approximation algorithm of Douglas and Peucker [13]. This algorithm also takes a maximum error ϵ as an input parameter. It starts with the first and the last breakpoint of the input function and then determines the point with the greatest distance to the line between the start and end point. If this distance is smaller than ϵ the error bound holds for all points and the two points are returned as the approximated function. If not, the point is inserted into the approximated function and the algorithm recurses on the partials from start to the inserted point and from inserted point to end. This algorithm may take $O(n^2)$ running time in the worst case.

[2]The result of linking two approximated travel time functions with known error bounds may have a much larger error depending on the slopes of the input functions. It would be possible to calculate the actual error bounds on the linked function at runtime, but TD-CRP does not attempt to do this. In practice, the errors stay small.

To obtain actual bound functions, we first compute an approximation. Then, we add or subtract ϵ to the value of each breakpoint to obtain an upper or lower bound, respectively. This yields valid upper or lower bounds, but they may not be as tight as possible. Therefore, we iterate over all approximated points and move each point back towards the original function. Both adjacent segments in the approximated functions have a minimum absolute error to the original function. We move the breakpoint by the smaller of the two errors. This yields sufficiently good bounds.

Preliminary experiments showed that compared to Imai-Iri, our Douglas-Peucker implementation produces insignificantly more breakpoints and also runs faster due to better constants. Also, the implementation needs 30 instead of 400 lines of code, so we use the Douglas-Peucker variant.

We approximate travel time functions whenever functions have more than β breakpoints after merging. This includes already approximated functions. Both β and the maximum error ϵ are tuning parameters. Note that the choice of ϵ does only affect performance and not correctness. We evaluate different choices in Section 5.1.

4.3 Queries

In this section, we present our query algorithms. We start by transferring the elimination tree query algorithm from [11] into the time-dependent setting. This approach suffers from bad performance since evaluating the travel time of a shortcut requires expensive unpacking. We then propose several optimizations which help to unpack fewer shortcuts and make the unpacking less expensive.

While the traditional CH query algorithm described in Section 2 can be applied to CCHs as well, the construction of the CCH makes a more efficient algorithm possible based on the *elimination tree*. In the elimination tree, the parent of each node is the lowest ranked upward neighbor in the augmented graph. The set of nodes in the upward search space from a node v is exactly the set of all ancestors of v in the elimination tree [3]. This allows for the following simplified query algorithm:

Starting from both s and t, walk up the elimination tree. For each node, relax all outgoing (incoming) arcs to (from) higher ranked neighbors. Once the searches meet at a common ancestor, track the total distance as the sum of forward and backward distances to the common ancestor. Once both directions reach the root node, the shortest distance has been found. This algorithm does not require a priority queue, but it will usually traverse more nodes than the traditional CH query. That makes it a great fit for long-range queries, but the performance on short-range queries is quite poor. Even when nodes are close to each other, all their ancestors up to the root are traversed and their arcs relaxed. In [6] an optimization is proposed to avoid this: only relax shortcuts from nodes when the node's distance is smaller than the tentative best distance between s and t. While this does not avoid traversing the entire tree, it avoids the relaxation of unnecessary arcs which is enough to achieve competitive running times on short range queries.

Naive time-dependent elimination tree query. To adapt the elimination tree query for the time-dependent case, a few modifications are necessary. First, to evaluate the travel time of shortcuts, we need to perform the time-dependent unpacking operation described in Algorithm 1 and evaluate the travel time of the unpacked path. Second, we do not know the arrival time at t and its ancestors. Thus, we cannot directly evaluate the travel time of their arcs. We avoid this problem by just marking all backward shortcuts in the search space of t. The evaluation is deferred and performed only after the forward search is done. The complete algorithm works as follows:

Starting from t, walk up the elimination tree. For each node, mark each edge to higher ranked neighbors and store it in a list at the neighbor. Once the elimination tree root is reached, switch to the forward direction. We maintain an earliest arrival label for each node, similar to Dijkstra's algorithm. The earliest arrival at s is initialized to the departure time. Starting from s, walk up the elimination tree. For each node, relax all shortcuts. Relaxation means evaluating the travel time as described in Section 3 and if possible improving the earliest arrival label at the shortcut's head. Once the root is reached, walk back down the path to t. For each node, relax all marked arcs. Having reached t, we know the desired earliest arrival at t.

Elimination tree interval query. The naive algorithm evaluates many shortcuts which do not contribute to the shortest path between s and t. We can use the bounds stored at our shortcuts to limit the search space to a subgraph, which contains the actual shortest path. We denote this subgraph as a *corridor*. The corridor is established through an *elimination tree interval query*. For an interval query, instead of maintaining an exact earliest arrival for each node, there is an interval of possible earliest arrivals. Several paths may contribute to the arrival interval at a node, so we need to maintain a set of labels for each node. Since we only work with bounds, backward shortcuts can be relaxed directly. The complete algorithm works as follows:

At each node, maintain an earliest arrival interval and a set of labels which contributed to the arrival interval. Each label contains the earliest arrival lower bound through its path and the id of the previous node on the path. Starting from s and t walk up the path to the elimination tree root. Walking both paths can be interleaved by always first processing the direction with the lower ranked current node. For each node, all shortcuts to higher ranked neighbors are relaxed. Relaxing a shortcut means constructing a corresponding label and updating the earliest arrival interval at the target node. If the labels lower bound on earliest arrival is earlier than the previous upper bound on earliest arrival, the label needs to be saved. Once the nodes earliest arrival interval is final, all labels with lower bounds greater than the upper bounds of the nodes earliest arrival interval can be removed. When both searches meet at a common ancestor, a tentative distance interval from s to t can be obtained. Common ancestors of s and t which contribute to the optimal distance interval are stored in a set of *meeting nodes*. Whenever the earliest arrival of a node is always later than the upper bound on the distance between s to t, we can skip shortcut relaxation entirely. Once both searches have reached the elimination tree root, we can construct the shortest-path corridor. We start by marking all meeting nodes as contained in the corridor. Then, the paths from s and t to the root are walked back down. For each node contained in the search space, we go over all labels and mark the corresponding shortcuts and nodes as contained in the search space. This yields the desired corridor.

Basic CATCHUp query. It would be possible to optimize the naive elimination tree query by first performing an interval query and limiting shortcut unpacking to shortcuts in the corridor. But this algorithm would still perform more work than necessary. When shortcuts partly share the same path, we may relax the shared path multiple times. We can avoid this by unpacking shortcuts only on demand. This query algorithm is inspired by the ATCH query algorithm presented in [2].

We start with the corridor obtained from the elimination tree interval query and run time-dependent Dijkstra. We modify the relaxation algorithm to gradually unpack shortcuts and expand the corridor by the unpacked parts. Algorithm 3 depicts this relaxation routine. If a relaxed shortcut directly represents an original arc, we know the exact travel time profile, and we can just relax the original arc. The earliest arrival of the head node of the original arc is then updated and if it was improved, the node will be inserted into the priority queue. If we encounter a shortcut that needs to be unpacked, we add the upward shortcut of the expansion to our corridor and recurse on the downward shortcut. Basically, we relax the first original arc on the path represented by the shortcut and mark all shortcuts on the path as contained in the search space. Once t is settled, we know the desired travel time from s to t.

Algorithm 3: ShortcutUnpackRelax

Input: Shortcut $s = (u, v)$ with unpacking data T_s, Time τ
Data: Priority queue Q, Tentative earliest arrivals ea, Upward shortcuts in corridor $C^\wedge$

$x_\tau \leftarrow x \in X_s$ such that $\tau \in \Pi_x$ `// binary search`

if x_τ *represents original arc* $(u, v) \in A$ **then**
  $\sigma \leftarrow \tau + \text{Eval}((u, v), \tau)$
  if $\sigma < \text{ea}(v)$ **then**
    $\text{ea}(v) \leftarrow \sigma$
    Q.InsertOrDecreaseKey$(v, \text{ea}(v))$
else
  $C^\wedge \leftarrow C^\wedge \cup (w_{x_\tau}, v)$
  ShortcutUnpackRelax$((u, w_{x_\tau}), \tau)$

The correctness of this approach was proven in [2]. While it achieves a good trade-off between preprocessing data and query speed, performance of long-range queries on complex graphs may still suffer. This is especially true for setup, since we only store upper and lower bounds for each shortcut and not upper and lower bound functions, which would have smaller errors. We now propose an additional extension to this algorithm which brings the performance of this algorithm almost up to speed with an exact TCH query. We call this query variant A*-extended ATCH query.

A-extended CATCHUp query.* Recall that A* changes the priority ordering based on a potential function p which estimates the remaining distance to t. The position of node v in the queue is determined by $\text{ea}(v) + \rho_t(v)$. In [17], it was shown that A* finds the correct shortest path, if for all arcs (u, v) $w_{(u,v)} - \rho_t(u) + \rho_t(v) >= 0$ holds, where $w_{(u,v)}$ is the static weight of the arc (u, v). If this condition holds, Dijkstra's algorithm is still label-setting and can be used without further modifications. Here, we are going to use a slightly weaker condition. In our case, a potential $\rho_{t,\sigma}$ for a target t and arrival time σ at t is valid, if for all nodes v $\rho_{t,\sigma}(v)$ is a lower bound of the actual distance of the reverse shortest-path tree to t arriving there at σ. This condition allows potentials which break Dijkstra's label-setting property. Nodes may be popped several times from the queue. But the condition still guarantees, that once t is popped from the queue, its distance is final. The reason for this is, that for any node v on the shortest path from s to t, the key of v in the queue must always be smaller than the key of t because the potential is a lower bound of the actual distance. Thus, all nodes on the shortest path will have been traversed before t is settled.

We are now going to reuse the results of the elimination tree interval query to obtain lower bounds on the distance to t for all nodes. During the elimination tree interval query, we obtained lower bounds for the remaining distance to t from all nodes in the upward search space of t. We can propagate these bounds to all other nodes we reach during our search and obtain lower bounds for them. First, we obtain lower bounds for all nodes in the upward search space of s. We use the lower bounds to t of the meeting nodes and propagate them downwards through the corridor to s by adding the lower bounds of the respective shortcuts. That yields lower bounds to t for all nodes in the corridor. We now extend our relaxation routine to derive potentials for nodes inserted into the corridor through unpacking. Algorithm 4 depicts this routine.

Algorithm 4: A*ExtendedShortcutUnpackRelax

Input: Shortcut $s = (u, v)$ with unpacking data T_s, Time τ
Data: Priority queue Q, Tentative earliest arrivals ea, Upward shortcuts in corridor $C^\wedge$

$x_\tau \leftarrow x \in X_s$ such that $\tau \in \Pi_x$ `// binary search`

if x_τ *represents original arc* $(u, v) \in A$ **then**
  $\sigma \leftarrow \tau + \text{Eval}((u, v), \tau)$
  if $\sigma < \text{ea}(v)$ **then**
    $\text{ea}(v) \leftarrow \sigma$
  if $\text{ea}(v) + \text{p}(v)$ *improved* **then**
    Q.InsertOrDecreaseKey$(v, \text{ea}(v) + \text{p}(v))$
else
  $\text{p}(w_{x_\tau}) \leftarrow \min(\text{p}(w_{x_\tau}), \text{p}(v) + \underline{b}_{(w_{x_\tau}, v)})$
  $C^\wedge \leftarrow C^\wedge \cup (w_{x_\tau}, v)$
  ShortcutUnpackRelax$((u, w_{x_\tau}), \tau)$

Compared to Algorithm 3, there are two modifications. First, the position of a node in the queue must be updated, when either its potential or its earliest arrival was improved. Second, when a shortcut needs to be expanded we derive a potential for the middle node w. For that, we take the potential of the shortcuts head v and add the lower bound of second part of the shortcut (w, v). This yields a lower bound for w, but only for paths to t over v. This may temporarily be an invalid potential if there is another shorter path from w over a node v' to t. Nevertheless, our algorithm achieves exact results.

Table 1: Characteristics of test instances used.

	Nodes [$\cdot 10^3$]	Arcs [$\cdot 10^3$]	TD arcs [%]	Avg. $\lvert f \rvert$ per TD arc	Size [GB]
Ger06	4 688	10 796	7	17.6	0.2
Ger17	7 248	15 752	29	29.6	0.7
Eur17	25 758	55 504	27	27.5	2.3

Consider all nodes on the actual shortest path. The ones in the initial corridor all have valid potential. Thus, they will be settled before t. When they are settled, the adjacent shortcuts will be unpacked, and unpacked nodes will get valid potentials. This happens before t is settled, so all nodes on the shortest path will have valid potentials before t is settled, so they will also be settled before t. Thus, when t is settled, the correct earliest arrival at t is known.

Since we perform a variant of Dijkstra's algorithm on the original graph, path unpacking is easy. During arc relaxation we maintain a set of parent pointers, which indicate the parent in the shortest path tree from s. The shortest path to t can be recalled by traversing the parents from t to the root s.

5 EXPERIMENTS

We implement our algorithms in Rust and compile them with rustc 1.36.0-nightly (372be4f36 2019-05-14) in the release profile with the target-cpu=native option[3]. To compile competing implementations written in C++, we use GCC 7.4. All experiments were conducted on a dual 8-core Intel Xeon Gold 6144 CPU with a base frequency of 3.5GHz with 192GiB of DDR4 RAM (clocked at 2.6GHz). We disabled hyperthreading and ran parallel experiments with 16 threads unless reported otherwise.

We use two production-grade instances for Germany and Europe and with traffic predictions for 2017. To compare our algorithms to related work, we also include an old instance of Germany from 2006. Traffic predictions are for a car on a typical midweek day. All graphs were provided by PTV. Table 1 lists key characteristics of each graph.

5.1 Customization

In this section, we investigate the performance of our customization phase using the Eur17 graph. Our best configuration ($\beta = 1000$, $\epsilon = 1.0s$)[4] with all optimization and 16 threads runs in about eight minutes (averaged over five runs). During the customization about 60GB of RAM are utilized. Travel time function approximation is a key factor to achieve this performance. Without approximation, the customization crashes since the available 192GB RAM are not enough. Table 2 depicts how different choices of ϵ and β influence the amount of breakpoints stored during the customization.

Clearly, both tighter approximation with a smaller ϵ and approximating less often due to a larger β lead to significantly greater memory consumption. Choosing a large β reduces the impact of ϵ since in that case approximation is performed only very seldom.

[3]We disabled AVX512 instructions for the customization, as they caused misoptimizations.

[4]We also evaluated relative error bounds, but they did not lead to any significant improvements or other noteworthy behavior.

Table 2: Average number of breakpoints per stored function sampled every three seconds during the customization of Eur17 with different approximation parameters.

ϵ	β				
	100	500	1 000	5 000	10 000
0.1s	3 434	3 571	3 639	3 978	5 130
0.5s	1 320	1 367	1 439	2 362	3 741
1.0s	978	1 068	1 157	2 101	3 517
5.0s	734	808	858	1 878	3 311
10.0s	656	712	789	1 842	3 275

Table 3: Customization running times in seconds on Eur17 with different approximation parameters.

ϵ	β				
	100	500	1 000	5 000	10 000
0.1s	1 500	1 480	1 479	1 211	1 109
0.5s	631	602	544	625	813
1.0s	629	529	480	575	774
5.0s	1 751	1 326	1 035	703	836
10.0s	3 493	2 621	1 893	885	907

On the other hand, choosing a small ϵ can make approximation ineffective when no further complexity reduction can be achieved, no matter how often approximation is performed.

When observing overall running time, the parameters also show a significant impact but the tendencies are not as unambiguous. While working with less complex function speeds up linking and merging on the approximation, larger sections will need to be unpacked, to perform exact merging. When approximating too roughly, this will be detrimental to the overall performance. Choosing a too large ϵ worsens performance significantly when β is small and approximation is performed often. For a larger β, the effect is less pronounced. On the other hand, choosing a small ϵ is also problematic since it achieves too little complexity reduction. For the Eur17 graph, $\epsilon = 1.0s$ and $\beta = 1000$ appears to be the sweet spot between complexity reduction and keeping enough information to avoid too much unpacking. This configuration also delivers good results for all other graphs. More careful parameter tuning might allow further reduction of customization times.

To evaluate their impact, we selectively disable triangle sorting and precustomization optimizations. Even though both optimizations exploit the same angle and speed-up customization by improving shortcut bounds to skip more linking and merging operations, they have each a significant impact on their own. Disabling the precustomization increases the overall customization running time to 647s. The effect of triangle sorting is even stronger. Disabling increases customization running time to 909s.

We also evaluate the effectiveness of our parallelization schema and run customization with a varying number of threads. Figure 5 depicts the results. As a baseline, we run the experiment with all parallelization code disabled. The running time is indicated by

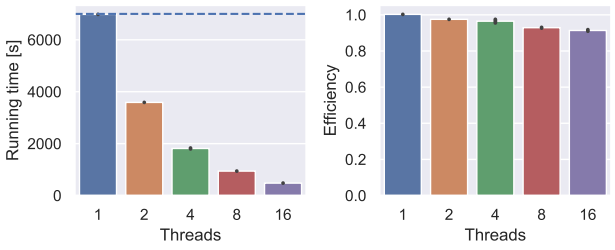

Figure 5: Average customization running times and parallelization efficiency (speedup/#threads) with standard deviations on five customization runs of Eur17. The dashed line indicates running time with all parallelization code disabled.

Table 4: Preprocessing output statistics. $|X|$ denotes the number of expansions per shortcut.

	CCH arcs $[\cdot 10^3]$	Avg. $\lvert X\rvert$	Max. $\lvert X\rvert$	$\lvert X\rvert = 1$ [%]
Ger06	22 521	1.075	44	98.4
Ger17	31 517	1.090	107	98.5
Eur17	115 023	1.100	115	98.4

the dashed line. Enabling parallelization but running with only one thread performs similarly. Nevertheless, running with more threads introduces some overhead due to synchronization. With 16 threads, parallel efficiency is still around 0.9. We conclude, that our implementation scales well and that customization times could be reduced further by running with more cores.

In Table 4, we report statistics about the data output by our preprocessing. On average, only 1.1 expansions per shortcut need to be stored for all graphs. About 98% of all shortcuts have only one expansion. The maximum number of expansions per shortcuts is only 115 even for our largest graph. This is still two orders of magnitude less than the number of breakpoints in the profile of that shortcut. This clearly shows the superiority of representing shortcuts through expansion information rather than explicitly storing travel time profiles.

5.2 Queries

We generate 100 000 source, target, departure time triples chosen uniformly at random. We use the same set of queries for our algorithms and available competing algorithms. Table 5 depicts the performance of our query algorithms in terms of running time and search space sizes averaged over all triples. Our A*-extended query algorithms require only few milliseconds to answer earliest-arrival queries, even on modern continental-sized production-grade instances. Without the A* optimization, queries take up to 5.5 times longer and the search space of the corridor Dijkstra grows roughly by the same factor. Both query variants relax only little more arcs than they settle nodes. The search space sizes of the interval queries differ only little between the old and the new Germany graphs. The reason for this is, that the interval query search space is determined primarily by the topology of the graphs. On the other hand, the search space sizes of the corridor Dijkstra clearly correspond to the complexity of the travel time functions of each graph.

We generate another set of queries to investigate the performance of our algorithms depending on the distance of source and target. We choose a start node and a departure time uniformly at random and perform time-dependent Dijkstra without a specific target. For every 2^ith settled node, we store source and target as a query of rank i. Figure 6 shows query running time of the basic query algorithm, the A*-extended query and path unpacking for queries with ranks from 7 to 24.

Obviously, query running times scale with the distance. Short range queries in both query variants usually take only fractions of milliseconds, except for outliers which take up to a millisecond. Short-range queries depict only a small advantage of the A*-extension. For long range queries, the advantage of the A*-extended query becomes much more significant. For the basic query, long range queries can take up to 100ms in extreme cases. The A*-extended query algorithm appears to be much more robust in those cases. Independent of the query rank, path unpacking takes about an order of magnitude less time than the actual query. For short-range queries, path unpacking times are basically negligible.

5.3 Comparison with related work

Finally, we compare our approach to other state-of-the-art approaches. Table 6 provides an overview over achieved preprocessing and query times, space overhead of the index data structures and average query errors where approximation is used. For KaTCH and TD-S, we could obtain the code and run experiments on our infrastructure. For all other techniques, we report their achieved numbers on the Ger06 graph in the configuration achieving the best query times. As experiments were performed on different machines, the numbers are not perfectly comparable. The newer graphs are significantly harder instances than Ger06. While many approaches perform decently on Ger06, it is unclear how much of that performance would still be achieved on the newer instances.

In our comparison, KaTCH achieves the best query times on Ger06 with slightly less than half a millisecond. CFLAT, heu SHARC and CATCHUp achieve comparable query times around half a millisecond. The original research implementation TCH reports slightly slower times. This may be because experiments were run on an older machine, but also because according to the KaTCH documentation, the newer query is somewhat more efficient. TCH pays for this speed with 4.7GB index data. Reducing the KaTCH memory consumption while keeping exactness (ATCH) brings query times up to 1.24ms. ATCHs also feature a configuration where they only keep upper and lower bounds for each profile (ATCH ∞). This configuration uses even less memory than CATCHUp because the optimized order results in fewer shortcuts. But this configuration degrades query performance even more. Giving up on exactness allows keeping the query times at 0.7ms (inex. TCH) but introduces some noticeable errors.

While achieving competitive query times for acceptable memory consumption, heu SHARC suffers from huge preprocessing times of several hours. The original publication does not report average query errors, only a maximum error of 0.61%. TDCALT has

Table 5: Detailed query performance and search space statistics for all graphs with and without our A* query extension. All values are averages over 100k random queries.

	Elimination tree interval query		Corridor unpacking Dijkstra					
	Nodes in CCH search space	Relaxed shortcuts	Queue pops		Relaxed arcs		Running time [ms]	
			Dij.	A*	Dij.	A*	Dij.	A*
Ger06	735	48 574	3 328	831	3 845	995	1.62	0.54
Ger17	770	58 903	18 499	3 099	19 986	3 502	8.70	1.59
Eur17	1 302	162 295	39 717	6 876	43 654	7 928	19.55	3.76

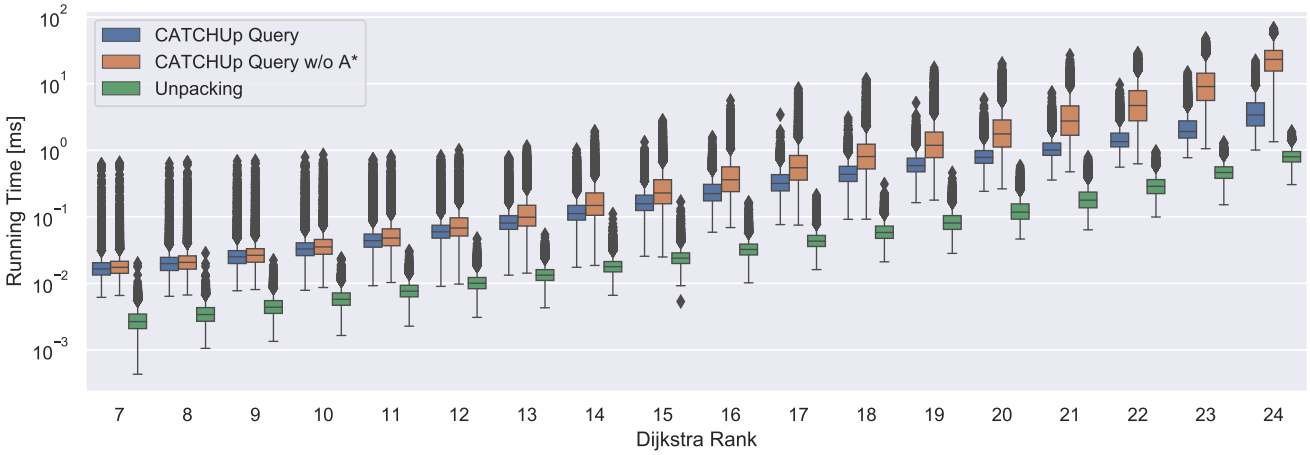

Figure 6: Query running times in milliseconds for queries of different Dijkstra rank on Eur17. The boxes cover the range between the first and the third quartile. The band in the box indicates the median. The whiskers indicate 1.5 times the inter quartile range. All other running times are marked as outliers.

the smallest memory consumption of all approaches but does not achieve competitive query times, even when approximating. FLAT and CFLAT both suffer from extreme preprocessing times and memory consumption despite having no exact queries. CATCHUp offers competitive query times for exact results while keeping memory consumption reasonable. Also, preprocessing is split in two phases and the metric could be exchanged within 18s on Ger06.

TD-CRP offers even lower memory consumption and a faster customization. But this is only possible through the use of approximation. TD-CRP queries depict a noticeable error and perform somewhat worse than KaTCH or CATCHUp queries. Utilizing an extremely simple approach, TD-S+9 depicts the smallest average error of all non-exact approaches[5]. TD-S can also be extended to support customizability, but no running times are reported. TCHs also support a faster preprocessing when reusing orderings. This could be used as a customization step, but running times are not competitive to TD-CRP and CATCHUp. Also, [5] reports that this approach falls apart when the new metric differs too much from the one used to calculate the ordering.

Path unpacking in the time-dependent scenario is not as easy as in the static setting. We only have separate numbers for KaTCH and CATCHUp where path unpacking on Ger06 takes up additionally to 200μs. For CFLAT, [25] report that path retrieval takes up to a third of the total query time. TD-CRP and FLAT do not support path retrieval.

On Ger17, KaTCH query times increase only very little. Memory usage on the other hand, grows almost by an order of magnitude. Path unpacking slows down by more than a factor three. For TD-S, both the growth in space consumption and query times corresponds roughly to the growth of the graph size, but not the increased number of breakpoints. For the index data of CATCHUp, this growth factor also roughly applies. Query times get about three times slower. Path unpacking on the other hand grows by less than a factor of two. While KaTCH clearly outperforms CATCHUp in terms of pure earliest-arrival query time, the difference becomes much less pronounced when looking at combined query and path unpacking times.

On Eur17, the memory consumption of KaTCH becomes prohibitive. While KaTCH is still able to finish preprocessing and output 150GB of data, queries crash since the 192GB RAM of our machine

[5] [25] report another CFLAT configuration with even smaller errors but significantly slower queries.

Table 6: Comparison with related work. We list unscaled numbers for algorithms we could not rerun ourself as reported in their respective publications. TD-Dijkstra, KaTCH and TD-S [30] were evaluated against the same set of instances as CATCHUp. Preprocessing times are averages over five runs. Query numbers are averages over the same set of 100k queries chosen uniformly at random. The EA column contains the time it takes to calculate the earliest arrival at the target node. The path column contains the additional time to retrieve the actual path. Values not reported are indicated as n/r, n/i states, that a feature was not implemented. OOM means that the program crashed while trying to allocate more memory than available. A similar overview with scaled numbers can be found in [10].

		Preprocessing		Customization		Index	Query			
		Time	Cores	Time	Cores	size	EA	Path	Rel. error	
		[s]		[s]		[GB]	[ms]	[ms]	Avg. [%]	Max. [%]
Ger06	TD-Dijkstra	-	-	-	-	-	525.48	n/r	-	-
	TDCALT [9]	540	1	-	-	**0.23**	5.36	n/r	-	-
	TDCALT-K1.15 [9]	540	1	-	-	**0.23**	1.87	n/r	0.050	13.840
	eco L-SHARC [7]	4 680	1	-	-	1.03	6.31	n/r	-	-
	heu SHARC [7]	12 360	1	-	-	0.64	0.69	n/r	n/r	0.610
	KaTCH	170	16	-	-	4.66	**0.43**	0.20	-	-
	TCH [2]	378	8	74	8	4.66	0.75	n/r	-	-
	ATCH (1.0) [2]	378	8	74	8	1.12	1.24	n/r	-	-
	ATCH (∞) [2]	378	8	74	8	0.55	1.66	n/r	-	-
	inex. TCH (0.1) [2]	378	8	74	8	1.34	0.70	n/r	0.020	0.100
	inex. TCH (1.0) [2]	378	8	74	8	1.00	0.69	n/r	0.270	1.010
	TD-CRP (0.1) [5]	273	16	16	16	0.78	1.92	n/i	0.050	0.250
	TD-CRP (1.0) [5]	273	16	**8**	16	0.36	1.66	n/i	0.680	2.850
	FLAT [25]	158 760	6	-	-	54.63	1.27	n/i	0.015	n/r
	CFLAT [25]	104 220	6	-	-	34.63	0.58	n/r	0.008	0.918
	TD-S+9	547	1	-	-	3.61	1.67	n/r	0.001	1.523
	CATCHUp	**31**	16	18	16	1.06	0.54	0.16	-	-
Ger17	TD-Dijkstra	-	-	-	-	-	869.79	n/r	-	-
	KaTCH	874	16	-	-	42.81	**0.71**	0.67	-	-
	TD-S+9	617	1	-	-	5.28	2.28	n/r	0.001	0.963
	CATCHUp	**35**	16	**92**	16	**1.50**	1.59	0.28	-	-
Eur17	TD-Dijkstra	-	-	-	-	-	2 581.16	n/r	-	-
	KaTCH	3 089	16	-	-	146.97	OOM	OOM	-	-
	TD-S+9	3 368	1	-	-	18.84	4.03	n/r	0.002	1.159
	CATCHUp	**196**	16	**479**	16	**5.48**	**3.76**	0.74	-	-

are not enough. Using ATCHs or inexact TCHs the memory consumption could likely be reduced sufficiently to perform queries. But this would either introduce errors or slow down queries significantly. On the other hand, with only 5.5GB of index data, CATCHUp is still able to perform exact queries in less than 4ms on average. This is fast enough to enable interactive applications. Total preprocessing for CATCHUp takes less than a quarter of the time KaTCH needs. TD-S+9 is also able to handle this instance with similar query times but only with a small average error.

6 CONCLUSION AND FUTURE WORK

We introduce CATCHUp – Customizable Approximated Time-dependent Contraction Hierarchies through Unpacking, a speed-up technique for routing in time-dependent road networks. It features a small index size and fast, exact queries. To the best of our knowledge, our approach is the first to simultaneously achieve all three objectives. Additionally, CATCHUp can incorporate updates to the metric within a few minutes even on modern continental-sized instances. We perform an in-depth experimental study to evaluate CATCHUp and compare it to competing approaches.

Revisiting ATCH, TCH, and TD-CRP with the insights gained in this work could be fruitful. Combining ATCH with our A*-extended query algorithm could reduce ATCH query running times. CATCHUp makes use of travel time independent node orders. Combining CATCHUp with TCH-like node orders could result in even smaller index sizes and query running times. We further expect that some of our optimizations to the customization routine can also be applied in a TD-CRP context. Another possible direction for future research would be to use a partial customization to further accelerate the customization phase. This could enable the integration of live traffic information.

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